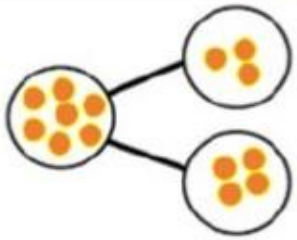
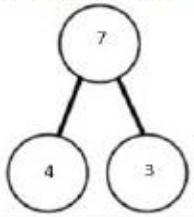
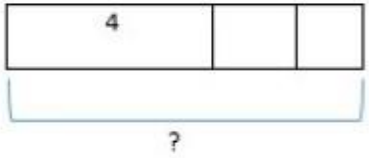
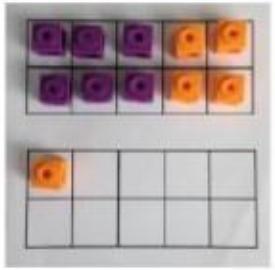
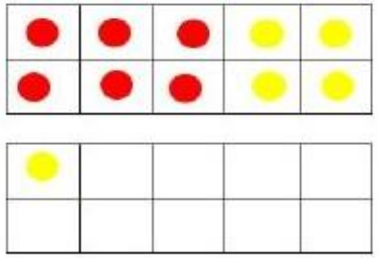
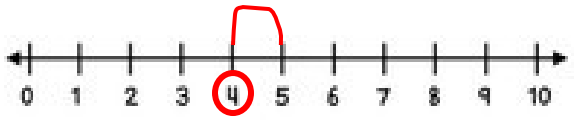
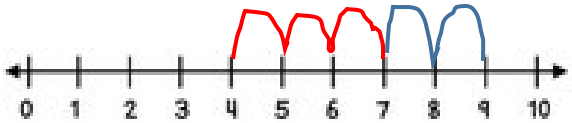
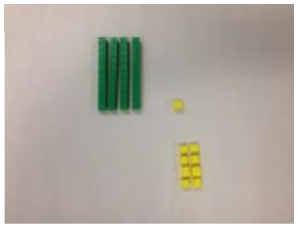
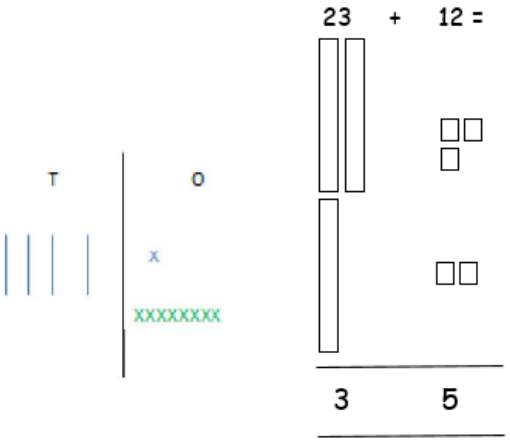
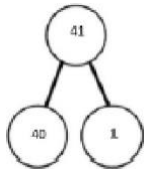
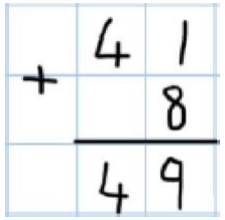
Toftwood Infant and Junior School Federation Calculation Procedures

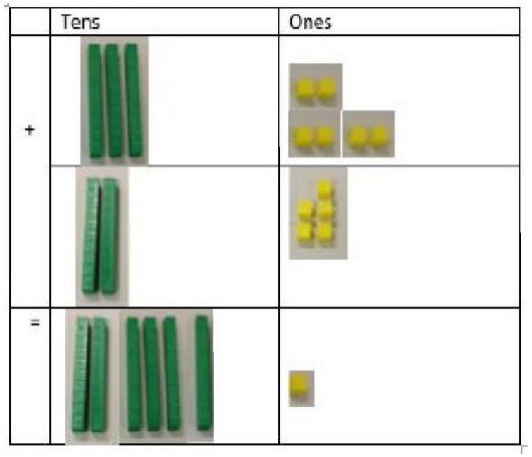
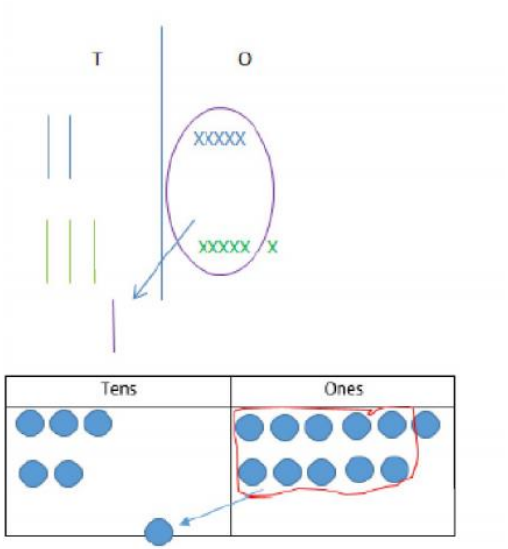
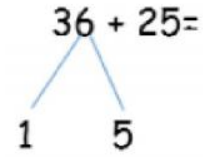
	EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Numicon Fingers Number line Objects/cubes Ten Frames	Numicon Fingers Number line Base 10 (concrete and pictorial)	Numicon Fingers Number line Base 10 (concrete and pictorial) Column method	Column method - regrouping. (up to 3 digits)	Column method - regrouping. (up to 4 digits)	Column method - regrouping. (with more than 4 digits) (Decimals - with the same amount of decimal places)	Column method - regrouping. (Decimals - with different amounts of decimal places)
Subtraction	Numicon Fingers Number line Objects/Cubes	Numicon Fingers Number line Base 10 (concrete and pictorial)	Numicon Fingers Number line Base 10 (concrete and pictorial)	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals - with the same amount of decimal places)	Column method with regrouping. (Decimals - with different amounts of decimal places)
Multiplication	Counting Fingers Doubling using dots Numicon	Counting Fingers Numicon Arrays Repeated Addition Using Multiplication Facts	Counting Fingers Cubes Arrays Repeated Addition Using Multiplication Facts	Counting in multiples Repeated addition Arrays - showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing using objects including cubes	Sharing into groups – using objects, pictorial representation Cubes	Sharing into groups – using objects, pictorial representation Grouping using pictorial representation Using multiplication facts	Division within arrays Division with a remainder Short division (2 digits by 1 digit concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number - interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number - interpret remainders as whole numbers, fractions or round)

Addition

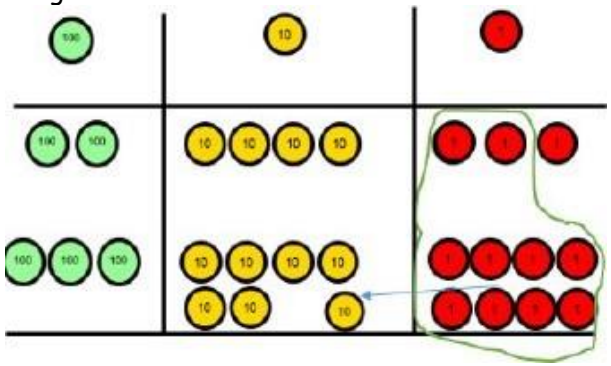
Key vocabulary which should be used: add, plus, total, altogether, more

Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears etc)  		4 + 3 = 7 (four is a part, 3 is a part and the whole is seven) 
Using Numicon to count on **4 + 3 = 7** 	A bar model which encourages the children to count on 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? 4 + 2 
Regrouping to make 10 by using ten frames and counters/cubes or using numicon: 6 + 5  	Children to draw the ten frame and counters/cubes 	Children to develop an understanding of equality e.g 6 + ☐ = 11 and 6 + 5 = 5 + ☐ 6 + 5 = ☐ + 4

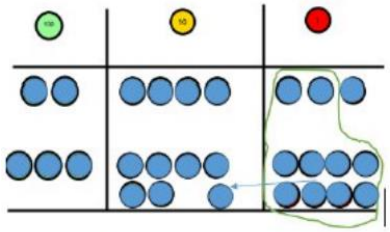
Using a number line to find one more than a given number.  **4 + 1 = 5** To add 3, one digit numbers  **2 + 3 + 4 = 9** Start on the largest number (4) then add the next biggest (3) then the last number (2). Moving onto the children identifying 3+2 = 5 and adding 4 and 5 to make 9.		
TO + O using base 10. Continue to develop understanding of partitioning and place value 41 + 8 	Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones or drawing the Base 10.  	41 + 8  1 + 8 = 9 40 + 9 = 49 

TO + TO using base 10. Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. 36 + 25  	This could be done one of two ways:  	Looking for ways to make 10  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: 36 +25 61 1

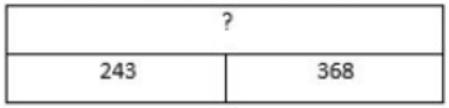
Use of place value counters add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract



Children to represent the counters e.g. like the image below

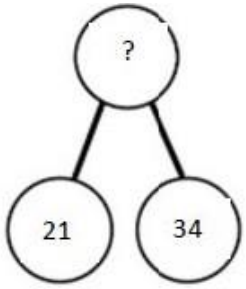


If the children are completing a word problem, draw a bar model to represent what it's asking them to do



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ \ 1 \end{array}$$

Fluency variation, different ways to ask children to solve 21 +34:



Sam saved £21 one week and £34 another. How much did he save in total?

21 + 34 = 55. Prove it! (reasoning but the children need to be fluent in representing this)

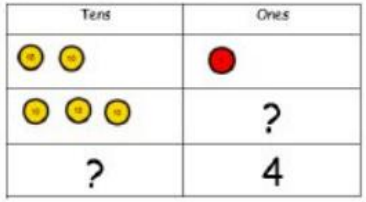
$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

What is the total of twenty one and thirty four?

21 + 34 =

□ = 21 + 34

Using the equals as a balance.



Identifying the missing digit.

<table>
<tr><td colspan="2">Children may use pictures/marks to aid their explanation or understanding, or to solve a mathematical concept/problem -</td><td>Solving missing number problems using one/two/three digits and link to number bonds –</td></tr>
<tr><td>I have 4 apples.
 
I have 4 apples. I find 2 more.

How many do I have now?
 
  </td><td>There are 16 pens in my bag and then I get 12 more. How many pens do I have altogether?

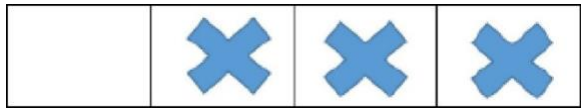
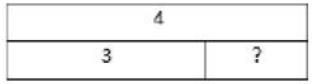
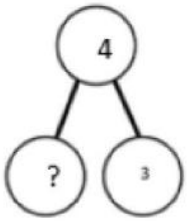
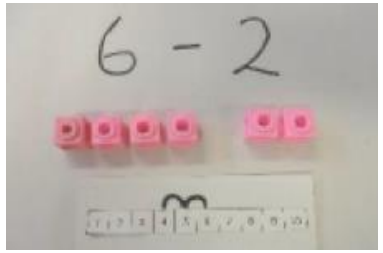
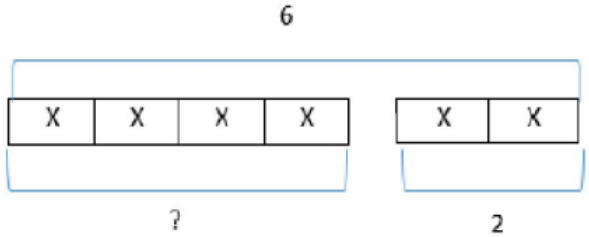
IIIII IIIII IIIII I + IIIII IIIII II</td><td>6 + ____ = 10 10 = ____ + 4

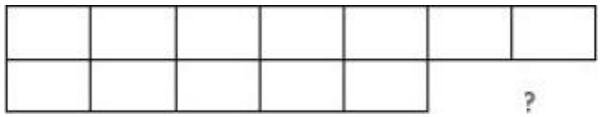
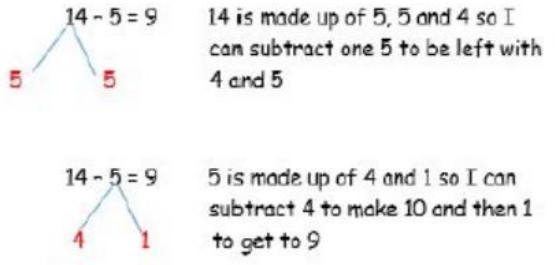
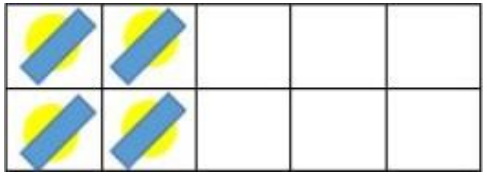
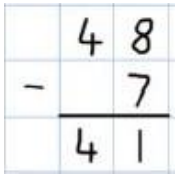
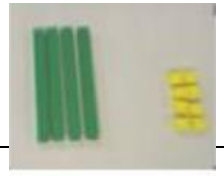
____ + 18 = 20 20 = 2 + ____

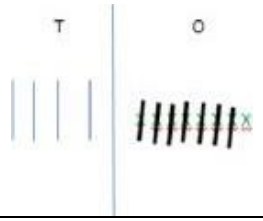
50 + ____ = 100 100 = 50 + ____</td></tr>
</table>

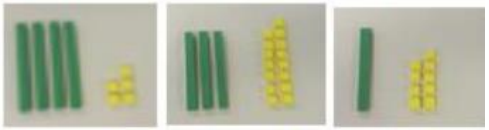
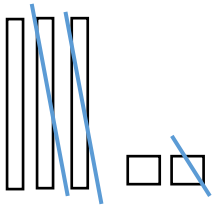
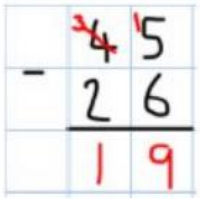
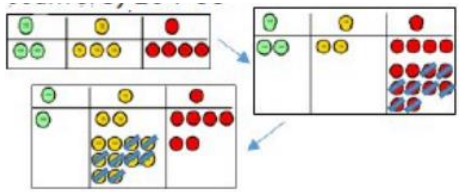
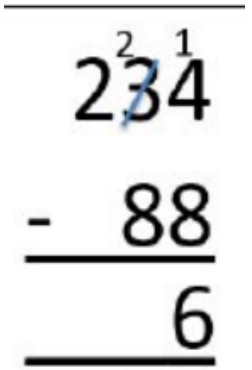
Subtraction

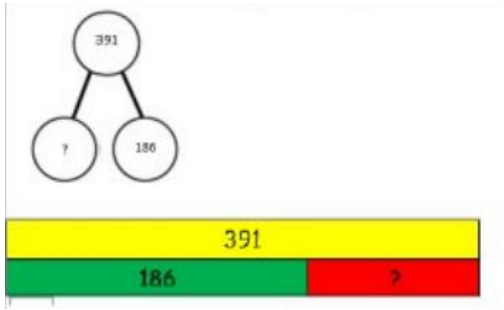
Key vocabulary which should be used: take (away), subtract, less, fewer

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (use various objects too) rather than crossing out children will physically remove the objects  	Children to draw the concrete resources they are using and cross out.  use of the bar model: 	4 - 3 = □ = 4 - 3  
Counting back using number lines.  Finding one less than a given number  **6 – 1 = 5**	Children to represent what they see pictorially e.g. 	Find the difference between 8 and 6. 8 - 6, the difference is ? Children to also explore why 9 - 7 = 8 - 6 (the difference, of each digit, has changed by 1 so the difference is the same - this will help when solving 10000-9987)

Finding difference (using cubes, Numicon other objects can also be used)  ? ? 	Children to draw the cubes/other concrete objects which they have used XXXXXXX XXXXX Use of the bar model  ?	14 – 5 = 9 You also want children to sec related facts e.g. 14 - 9 = 5 Children to represent how they have solved it e.g. 
Making 10 (using numicon or ten frames) 14 - 5  Children could also do this by subtracting a 5 from the 10. 	Children to present the ten frame pictorially 	48 - 7 = 
Column method (using base 10) 48-7 	Represent the base 10 pictorially	

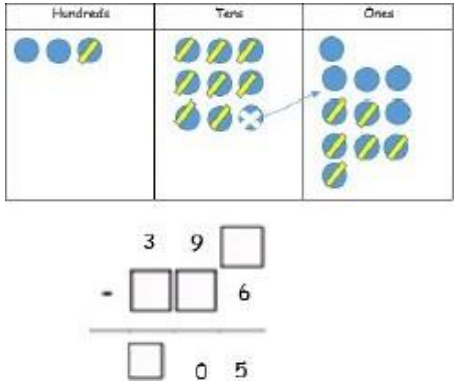
Column method (using base 10 and having to exchange)  1) Start by partitioning 45 2) Exchange one ten for ten more ones 3) Subtract the ones, then the tens	Represent the base 10 pictorially 32 – 21 = 	It's crucial that the children understand that when they have exchanged the 10 they still have 45. 45 – 26 = 19 
Column method (using place value counters) 234-88 	Once the children have had practice with the concrete, they should be able to apply it to any subtraction. Like the other pictorial representations, children represent the counters.	
Fluency variation, different ways to ask children to solve 391-186:		

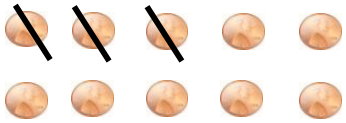
<table>
<tr>
<td> 
</td>
<td>Raj spent £391, Timmy spent £186. How much more did Raj spend?

I had 391 metres to run. After 186 1 stopped. How many metres do I have left to run?</td>
<td>391 - 186

☐ = 391 - 186

391
-186
____

Find the difference between 391 and 186
Subtract 186 from 391.
What is 186 less than 391?</td>
<td>What's the calculation? What's the answer?
 
</td>
</tr>
<tr>
<td colspan="3">Children may use pictures/marks to aid their explanation or understanding, or to solve a mathematical concept/problem -</td>
<td>Solving missing number problems using one/two/three digits and link to number bonds –
Children understand commutativity and that the order of numbers in a subtraction calculation is important.</td>
</tr>
<tr>
<td>I had 10p. I spent 3p. How much do I have left?

  </td>
<td>There are 14 apples in a bag. Jim took 3.

How many are left?

  </td>
<td colspan="2">10 - _ = 7

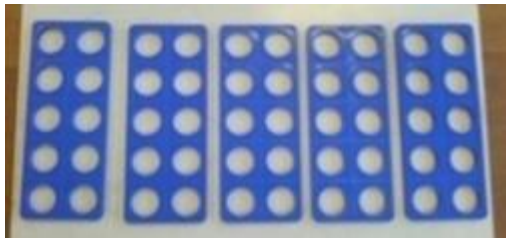
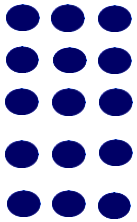
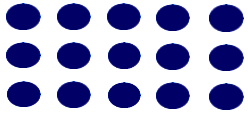
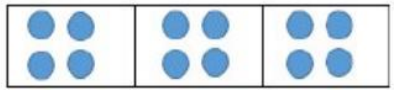
10 – 3 = _

_ - 7 = 3

_ - 3 = 7</td>
</tr>
</table>

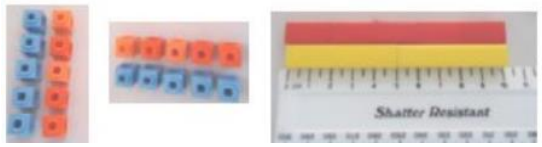
Multiplication

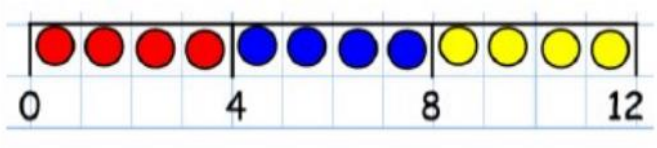
Key vocabulary which should be used: multiply, repeated addition, groups, equal, pattern

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition (using cubes, Numicon and other objects)  4 x 3 or 4, 3 times  10 x 5 or 10, 5 times	Children to represent the practical resources in a picture Doubling numbers (related to multiplying by 2)  As arrays  3 x 5 = (3, 5 times)  5 x 3 = (5, 3 times) Use of a bar model for a more structured method 	4 x 3 4 + 4 + 4

<table>
<tr>
<td>Use arrays to illustrate commutativity (counters and other objects can also be used)

2 x 5 = 5 x 2

  </td>
<td>Represent this pictorially alongside a number line e.g.:

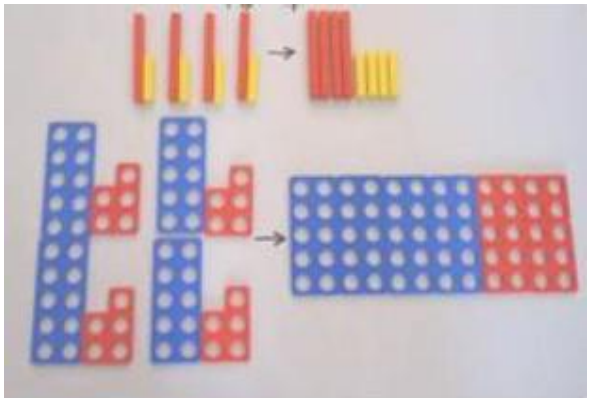
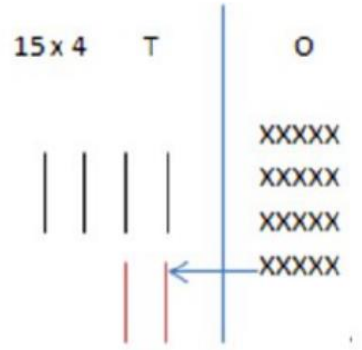
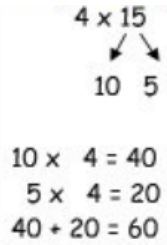
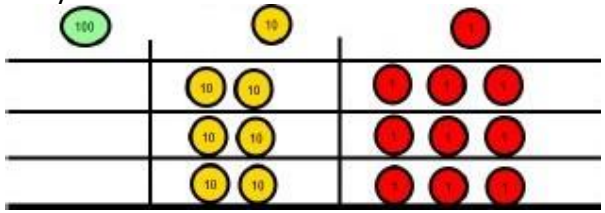
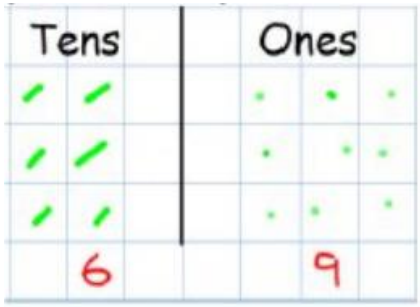
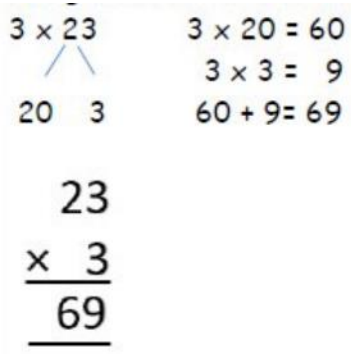
 

Children are encouraged to use multiplication facts to solve calculations and may draw marks/pictures or use their fingers to support this –

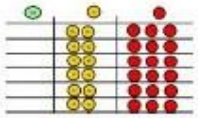
2, 4, 6, 8, 10, 12, 14

|| || || || || || ||</td>
<td>Children to be able to use an array to write a range of calculations e.g.

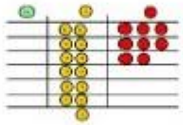
2 x 5 = 10
5 x 2 = 10
2 + 2 + 2 + 2 + 2 = 10
5 + 5 =10</td>
</tr>
</table>

Partition to multiply (use numicon, base 10, Cuisenaire rods) 4 x 15 	Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like: 15 x 4  	Children to be encouraged to show the steps they have taken 
Formal column method with place value counters or base 10 (at the first stage - no exchanging) 3 x 23 Make 23, 3 times. See how many ones, then how many tens 	Children to represent the counters pictorially:  	Children to record what it is they are doing to show understanding 
Formal column method with place value counters (children need this stage, initially, to understand how the column method works)		**6 x 23** 6 x 3 = 18 6 x 20 = 120 120 + 18 = 138

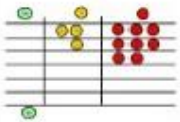
6 x 23



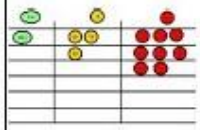
Step 1: get 6 lots of 23



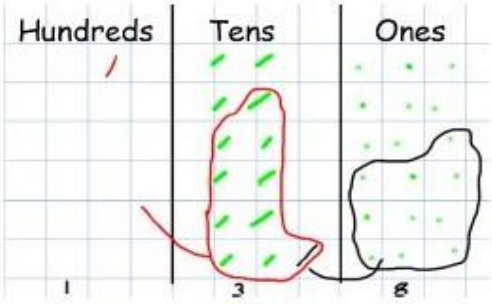
Step 2: 6 x 3 is 18. Can I make an exchange? Yes! Ten ones for one ten….



Step 3: 6 x 2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred…



Step 4- what do I have I each column?



The aim is to get to the formal method but the children need to understand how it works.

$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

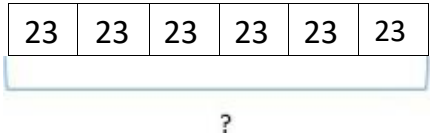
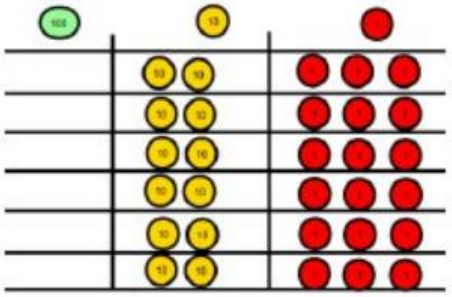
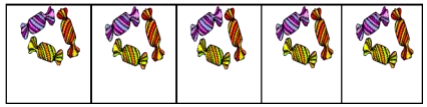
When children start to multiply 3d x 3d and 4d x 2d etc, they should be confident with the abstract:

To get 744 children have solved 6 x 124
To get 2480 they have solved 20 x 124

$$\begin{array}{r} 1\ 2\ 4 \\ \times\ \ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 1 \quad 2 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ \ 1 \end{array}$$

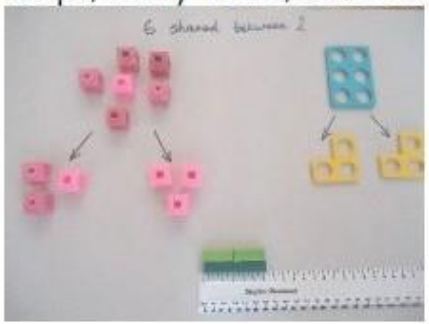
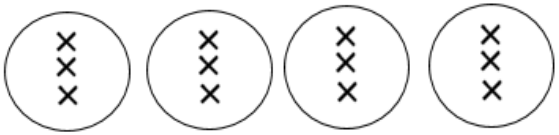
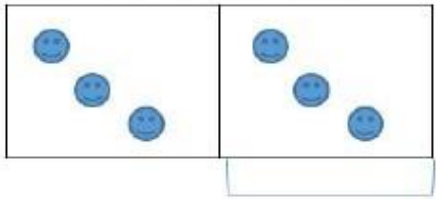
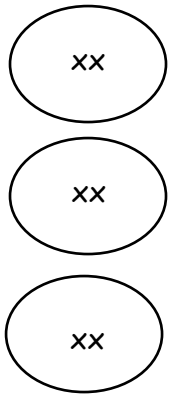
Answer: 3224

Fluency variation, different ways to ask children to solve 6 x 23:

 With the counters, prove that 6 x 23 = 138 Why is 6 x 23 = 23 x 6?	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? Tom saved 23p three days a week. How much did he save in 2 weeks?	Find the product of 6 and 23 6 x 23 = □ = 6 x 23 6 × 23 ___ 23 × 6 ___	What's the calculation? What's the answer? 
Children may use pictures/marks to aid their explanation or understanding, or to solve a mathematical concept/problem -			Solving problems using multiplication facts including missing number problems -
There are 6 flowers in each pot. How many in 2 pots? 	There are 3 sweets in each box. How many in 5 boxes? 		Each bag has 5 apples in it. How many apples in 6 bags? **5, 10, 15, 20, 25, 30** 3 x 5 = __ 15 = 5 x __ 3 x _ = 15 15 = 3 x _ _ x 5 = 15 _ = 3 x 5 _ x _ = 15 15 = _ x _

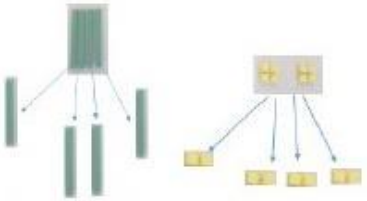
Division

Key vocabulary which should be used: share, group(s), divide, equal

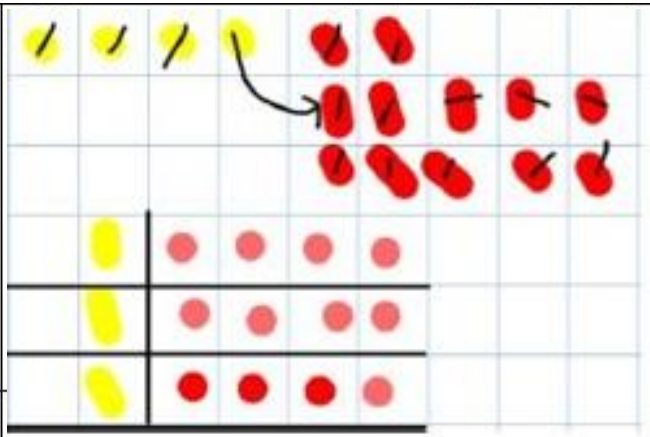
Concrete	Pictorial	Abstract
6 shared between 2 (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates) 	12 ÷ 4 =  This can also be done in a bar so all 4 operations have a similar structure: 	6 ÷ 2 = 3 What's the calculation? 3 | 3
Understand division as repeated grouping and subtracting 6 ÷ 2 		Children are encouraged to use their multiplication facts to solve division calculations – 30 ÷ 10 = 10, 20, 30
2d ÷ 1d with remainders 13 ÷ 4 = 3 remainder 1	Children to have chance to represent the resources they use in a pictorial way e.g. see below:	13 ÷ 4 = 3 remainder 1 Children to count their times tables facts in their heads

2d divided by 1d using base 10 (no remainders) SHARING

48 ÷ 4 = 12



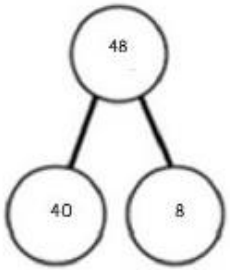
Start with the tens.



48 ÷ 4

4 tens ÷ 4 = 1 ten

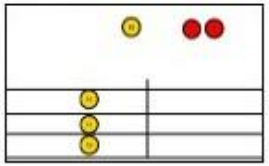
8 ones ÷ 4 = 2 ones



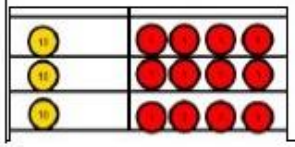
10 + 2 = 12

Sharing using place value counters.

42 ÷ 3= 14



1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10?



Exchange the ten for 10 ones and share out 12 ones

42 ÷ 3

42 = 30 + 12

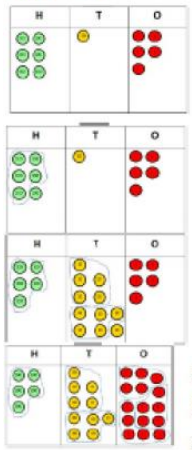
30 ÷ 3 = 10

12 ÷ 3 = 4

10 + 4 = 14

Use of 'bus stop method' using grouping and counters. Key language for grouping - how many groups of X can we make with X hundreds'- this can also be done using sharing!

615 ÷ 5



Step 1: make 615

Step 2: Circle your groups of 5

Step 3: Exchange 1H for 10T and circle groups of 5

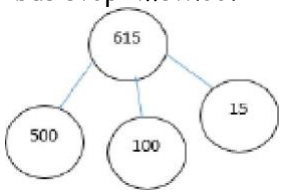
Step 4: exchange 1T for 10ones and circles groups of 5

This can easily be represented pictorially, until the children no longer need to do it.
It can also be done to decimal places if you have a remainder!

$$5\overline{)615}\quad 123$$

Fluency variation. different ways to ask children to solve 615 + 5:

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

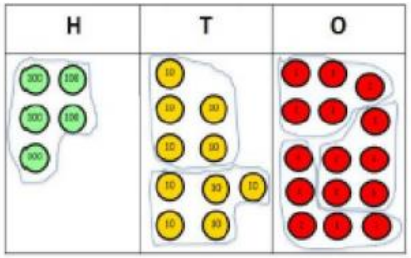
$5\overline{)615}$

615 ÷ 5 =

□ = 615 ÷ 5

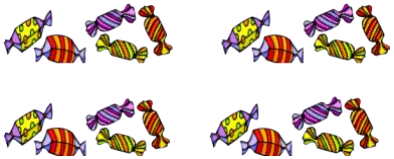
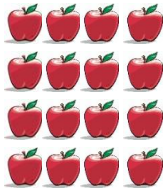
How many 5's go into 615?

What's the calculation? What's the answer?



<table>
<tr><td colspan="3">Children may use pictures/marks to aid their explanation or understanding, or to solve a mathematical concept/problem</td><td>Solving problems using multiplication facts including missing number problems.

Children understand commutativity and that the order of numbers in a division calculation is important.</td></tr>
<tr><td>Ten children are split into 2 groups.

How many children in each group?
  </td><td>There are 20 sweets. They are shared between 4 people. How many do they have each?
  </td><td>There are 16 apples in a basket. They are shared between 4 people. How many do they have each?
  </td><td>Ben has 12 pens and shares them between two pots. How many in each pot?

2, 4, 6, 8, 10, 12

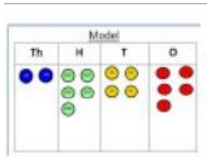
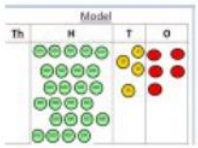
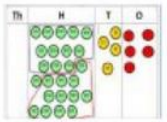
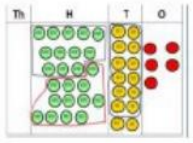
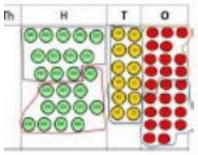
1 2 3 4 5 6

15 ÷ 3 = __ 5 = 15 ÷ __

_ ÷ 5 = 3 _ = 15 ÷ 5

_ ÷ _ = 3 3 = _ ÷ _</td></tr>
</table>

Long division

Concrete	Pictorial	Abstract
 0212 12 2544 24 14 12 24 24 0 2544 ÷ 12 How many groups of 12 thousands do we have? None	Children to represent the counters, pictorially and record the subtractions beneath.	0 12 2544 Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.
 Exchange 2 thousand for 20 hundreds.		02 12 2544 24 1 Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.
 02 12 2544 24 1 How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.		021 12 2544 24 14 12 2 Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.
 021 12 2544 24 14 12 2 Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.		0212 12 2544 24 14 12 24 24 0 Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.
 Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2		